

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Advanced Calculus

Subject Code: 4SC03ADC1

Branch: B.Sc. (Mathematics)

Semester: 3

Date: 22/11/2022

Time: 11:00 To 02:00

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1 Attempt the following questions: (14)**
- a) True or False If $f'(x) > 0 \forall x \in (a, b)$ then the f is said to be an decreasing function on interval (a, b) . (01)
 - b) Define: Point of Inflexion. (01)
 - c) Define: Beta function. (01)
 - d) The value of $\Gamma\left(\frac{1}{2}\right)$ is_____. (01)
 - e) What is the relation between Beta function and Gamma function? (01)
 - f) find $\lim_{\substack{x \rightarrow 5 \\ y \rightarrow -2}} x^3 + 4x^2y - 5xy$ (01)
 - g) If $u = y^x$ then find $\frac{\partial u}{\partial x}$. (01)
 - h) The degree of homogeneous function $f(x, y) = x^2 + 2xy + y^2$ is_____. (01)
 - i) Find the interval in which the function $x^2 - 4x + 3$ is increasing. (02)
 - j) If $u = e^{xy}$ then find $\frac{\partial^2 u}{\partial x^2}$. (02)
 - k) Find the value of $\beta(3,4)$. (02)

Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions (14)**
- a) Find the intervals in which the function $f(x) = 2x^3 + 3x^2 - 36x - 12$ is increasing or decreasing. (05)
 - b) Find the interval in which the curve $y = x^4 - 4x^3$ concave up and concave down. (05)
 - c) Find the points of inflection on the curve $f(x) = x^2 - 5x + 6$. (04)
- Q-3 Attempt all questions (14)**
- a) Prove Symmetric property of Beta function. (05)



- b) Show that $\beta(p, q) = 2 \int_0^{\frac{\pi}{2}} \sin^{2p-1} \theta \cos^{2q-1} \theta d\theta.$ (05)
- c) Prove that $\Gamma \frac{1}{2} = \sqrt{\pi}.$ (04)

Q-4 Attempt all questions (14)

- a) Evaluate: $\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 2}} \frac{x+y}{x-y}$ and $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x-1}{y-1}$ (05)
- b) Evaluate: $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^3 - y^3}{x^2 + y^2}$ (05)
- c) Evaluate: $\lim_{\substack{x \rightarrow 2 \\ y \rightarrow 3}} \frac{x^2 + y^3}{2x^2y}$ (04)

Q-5 Attempt all questions (14)

- a) If $u = e^{xyz}$ then prove that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}.$ (05)
- b) If $Z = \frac{x^2 + y^2}{x + y}$ then show that $\left(\frac{\partial Z}{\partial x} - \frac{\partial Z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial Z}{\partial x} - \frac{\partial Z}{\partial y}\right).$ (05)
- c) If $u = \tan^{-1} \frac{y}{x}$ then find $\frac{\partial^2 u}{\partial x \partial y}.$ (04)

Q-6 Attempt all questions (14)

- a) State and prove Euler's first theorem for homogeneous function. (05)
- b) If $u = \sin^{-1} \frac{x}{y} + \cos^{-1} \frac{y}{z} + \tan^{-1} \frac{z}{x}$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ (05)
- c) If $f(x, y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2}$ then show that (04)

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} - 2f(x, y) = 0.$$

Q-7 Attempt all questions (14)

- a) Find the intervals in which $f(x) = x^4 - 6x^3$ is concave upward and concave downward. (05)
- b) Evaluate: $\int_0^{\frac{\pi}{2}} \sin^5 \theta \cos^6 \theta d\theta.$ (05)
- c) Show that $\frac{1}{5} \beta\left(\frac{2}{5}, \frac{1}{5}\right) = \int_0^{\infty} \frac{x}{\sqrt{1-x^5}} dx.$ (04)



Q-8

Attempt all questions

(14)

a) If $u = e^{x^2+y^2+z^2}$ then show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = 8xyz(e^{x^2+y^2+z^2})$. (06)

b) If $x^3 + y^3 + z^3 - 3xyz = 0$ then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. (04)

c) Evaluate $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2}$. (04)

